Shore School

Exam Number:

Set:

2016

Year 12 Trial HSC Examination

Mathematics Extension 1

General Instructions

- Reading time − 5 minutes
- Working time – 2 hours
- Write using black pen
- Board-approved calculators may be used
- A reference sheet is provided
- In Questions 11–14, show relevant mathematical reasoning and/or calculations
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks – 70

Section I Pages 2–5

10 marks

- Attempt questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–13

60 marks

- Attempt Questions 11–14
- Allow about 1 hour 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1−10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1−10.

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1 Which of the following does NOT have an inverse function?

(A) $y=\sqrt{x}$

(B) $y=x^2$

(C) $x=\sqrt{y}$

(D) $x=y^2$

2 What is $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{3}}{5x}$?

(A) $\dfrac{1}{15}$

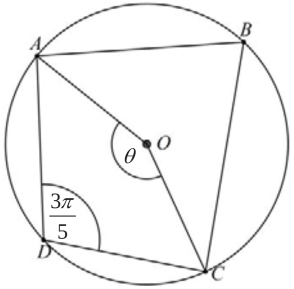
(B) $\dfrac{3}{5}$

(C) $\dfrac{5}{3}$

(D) 15

3 $ABCD$ is a cyclic quadrilateral, with all vertices lying on a circle with centre O.

The size of $\angle ADC$ is $\frac{3\pi}{5}$ radians.



NOT TO SCALE

What is the size of angle θ?

(A) $\frac{\pi}{5}$

(B) $\frac{2\pi}{5}$

(C) $\frac{3\pi}{5}$

(D) $\frac{4\pi}{5}$

4 The point P divides the interval from $A(4,-2)$ to $B(6, 2)$ externally in the ratio 5 : 3.

What are the coordinates of P?

(A) $(1,-8)$

(B) $\left(\frac{19}{4},-\frac{1}{2}\right)$

(C) $\left(\frac{21}{4},\frac{1}{2}\right)$

(D) $(9, 8)$

5 The parametric form of a parabola is $(3t,-6t^2)$.

What is the Cartesian form of this parabola?

(A) $x^2=-12y$

(B) $x^2=-\frac{3}{2}y$

(C) $x^2=\frac{3}{2}y$

(D) $x^2=12y$

6 A family of 8 sit at a circular table.

What is the probability that the two youngest children **do not** sit next to each other?

(A) $\frac{5}{7}$

(B) $\frac{3}{4}$

(C) $\frac{20}{21}$

(D) $\frac{2519}{2520}$

7 Suppose θ is the acute angle between the lines $y=3x-1$ and $x+2y-6=0$.

What is the value of $\tan\theta$?

(A) -7

(B) -1

(C) 1

(D) 7

8 What is the domain of $y = \cos^{-1}\left(\frac{3x}{2}\right)$?

(A) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

(B) $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

(C) $-\frac{2}{3} \le x \le \frac{2}{3}$

(D) $-\frac{3}{2} \le x \le \frac{3}{2}$

9 When the polynomial $P(x) = ax^3 - 5x^2 + 10x + 12$ is divided by $(x-2)$ the remainder is 4.
What is the remainder when $P(x) = ax^3 - 5x^2 + 10x + 12$ is divided by $(x+3)$?

(A) –36

(B) –30

(C) –4

(D) –1

10 If $y = \cos(\ln x)$, which of the following is the correct expression for $\frac{d^2y}{dx^2}$?

(A) $\frac{d^2y}{dx^2} = -\cos(\ln x)$

(B) $\frac{d^2y}{dx^2} = \frac{-\cos(\ln x)}{x^2}$

(C) $\frac{d^2y}{dx^2} = \frac{\sin(\ln x) - \cos(\ln x)}{x^2}$

(D) $\frac{d^2y}{dx^2} = \frac{-\sin(\ln x)}{x}$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Consider the polynomial $P(x) = 6x^3 - 5x^2 - 13x + 12$, with roots α, β and γ. **2**

Determine the value of $\alpha^2 + \beta^2 + \gamma^2$.

(b) How many arrangements of the letters of the word PARRAMATTA are possible? **2**

(c) Solve the inequality $\frac{5}{x-4} > x$. **3**

(d) Use the substitution $u = e^x$ to evaluate $\int_0^{\ln\frac{1}{\sqrt{2}}} \frac{5e^x}{\sqrt{1-e^{2x}}}\,dx$. **3**

Leave your answer in exact form.

(e) Differentiate $x^2 \cos^{-1} x$. **2**

(f) Consider the binomial expansion of $\left(2x^3 - \frac{3}{x}\right)^{12}$. **3**

Find an expression for the term independent of x.

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) Sean took a piece of beef out of his freezer which was set at –20ºC and placed it straight into an oven which was preheated to 160ºC. The temperature at the centre of the meat may be modelled using the formula

$$T = 160 - Ae^{-kt},$$

where A and k are constants, and t is the time in minutes after the piece of beef was put in the oven.

(i) After 50 minutes, Sean used his thermometer to discover that the beef was 40ºC at its centre. **2**

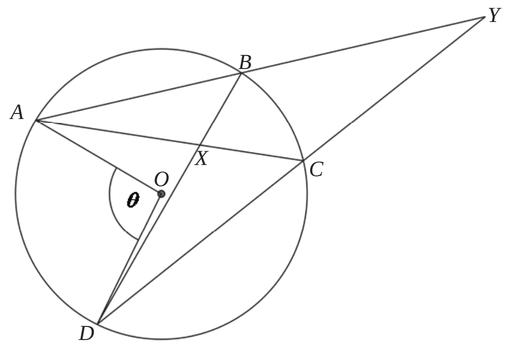
Show that $k = \frac{1}{50}\ln\left(\frac{3}{2}\right)$.

(ii) Sean wants his beef to be cooked 'medium rare' which requires it to be 60ºC at its centre. **2**
How long after he put the beef in the oven, should Sean take it out?
Give your answer correct to the nearest minute.

(b) Ted tosses **two** unbiased coins in 10 separate trials. **2**
Write an expression for the probability of both coins landing 'heads' in exactly 6 of these trials.

Question 12 continues on page 8

(c) In the diagram, A, B, C and D are points on a circle centre O, and $\angle AOD = \theta$.
The lines AB and DC intersect at Y and the lines AC and DB intersect at X.

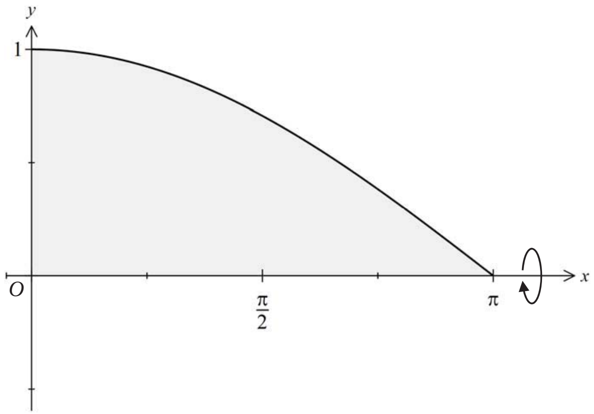


(i) Explain why $\angle DBY = 180° - \frac{\theta}{2}$. **1**

(ii) Show that $\angle BXC + \angle BYC = \theta$. **2**

Question 12 continues on page 9

(d) The region bounded by the curve $y = \cos\frac{x}{2}$, the x-axis, and the y-axis is rotated about the x-axis to form a solid. **3**



Find the volume of this solid of revolution.

(e) Prove by mathematical induction that for all integers $n \geq 2$, **3**

$$n^2 \geq 2n.$$

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) Consider the even function $f(x) = \frac{2x^2+9}{x^2+4}$.

(i) Prove that $f(x) = 2 + \frac{1}{x^2+4}$. **1**

(ii) Hence, or otherwise, state the equation of the horizontal asymptote of the curve $y = f(x)$. **1**

(iii) Find the coordinates of the stationary point on $y = f(x)$. **1**

(iv) Hence, sketch $y = f(x)$, showing the features found above. **2**

(v) Find the area bounded by the curve $y = f(x)$, the line $x = -2$, the line $x = 2$, and the x-axis. **2**

(b) A particle is moving in a straight line according to the equation

$$x = \sqrt{3}\sin 2t + \cos 2t$$

where x is the displacement in metres and t is the time in seconds.

(i) Determine the value of R and α, for which $x = R\sin(2t+\alpha)$, with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **2**

(ii) Prove that the particle is moving in simple harmonic motion by showing that x satisfies an equation of the form $\ddot{x} = -n^2 x$. **3**

(iii) Find when the particle is first stationary and its displacement at that time. **2**

(iv) Hence, or otherwise, find the distance travelled by the particle in the first π seconds. **1**

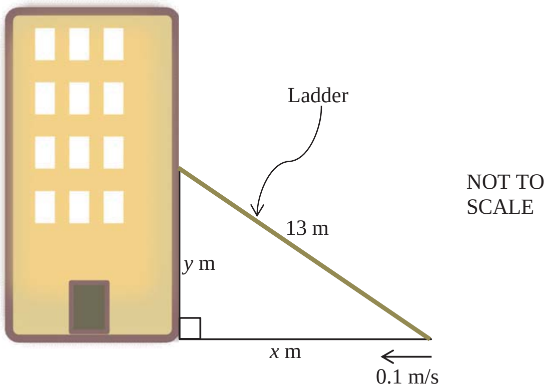
Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) The equation $x^3 = \cos x$ has only one solution, which is near $x = 0.9$. **2**
Use one application of Newton's method to find a better approximation to the solution.
Leave your answer correct to 2 decimal places.

(b) Find the general solution(s) of $4\sin\frac{x}{2}\cos\frac{x}{2} = \sqrt{3}$. **2**

Leave your answer in radians.

(c) A fireman placed his 13 metre long ladder against a wall as shown in the diagram below. **2**



As it didn't reach up high enough, he started pushing the foot of the ladder closer to the wall at the rate of 0.1 m/s.

Find the rate at which the ladder is moving higher on the wall when the foot of the ladder is 5 metres out from the base of the wall.

Question 14 continues on page 12

(d) The equations of motion of a projectile fired from the origin with initial velocity V ms^{-1} at angle θ to the horizontal are

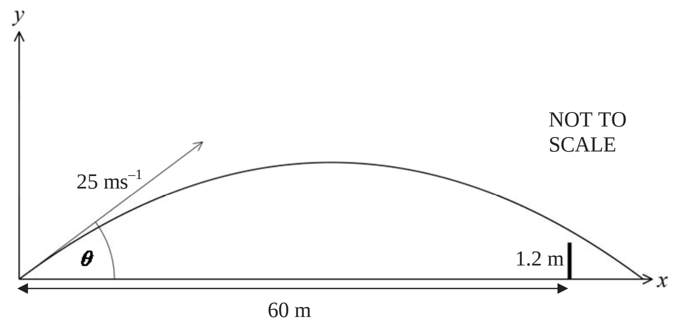
$$x = Vt\cos\theta \quad \text{and} \quad y = -\frac{1}{2}gt^2 + Vt\sin\theta. \quad \text{(Do NOT prove this.)}$$

A cricket ball was hit from a point on horizontal ground at a speed of 25 ms^{-1}.

You may assume that $g = 9.8$ ms^{-2}.

(i) Prove that the flight path of the cricket ball is given by **2**
$y = -\frac{49x^2}{6250}(1 + \tan^2\theta) + x\tan\theta$.

(ii) The cricket ball cleared a 1.2 metres high boundary fence 60 metres from where the ball was hit. **3**



Determine the possible values of θ.

Question 14 continues on page 13

(e) Consider the binomial expansion

$$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + {}^nC_4x^4 + \ldots + {}^nC_{n-1}x^{n-1} + {}^nC_nx^n .$$

(i) Find an expression for **1**

$${}^nC_0x^{-1} + {}^nC_1 + {}^nC_2x + {}^nC_3x^2 + {}^nC_4x^3 + \ldots + {}^nC_{n-1}x^{n-2} + {}^nC_nx^{n-1} .$$

(ii) By differentiating, prove that **3**

$${}^nC_2 + 2\,{}^nC_3 + 3\,{}^nC_4 + \ldots + (n-2)\,{}^nC_{n-1} = (n-2)(2^{n-1} - 1) .$$

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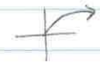
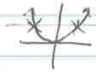
Yr 12 Maths Ext 1 Trial Solutions 2016

① B

$y = \sqrt{x}$  $y = x^2$ 

$x = \sqrt{y}$  $x = y^2$ 

② A

$$\lim_{x\to 0} \frac{\sin \frac{x}{3}}{5x} = \frac{1}{15} \times \lim_{x\to 0} \frac{\sin\frac{x}{3}}{\frac{x}{3}}$$
$$= \frac{1}{15} \times 1$$
$$= \frac{1}{15}$$

③ D

$\angle ABC = \pi - \frac{3\pi}{5}$ (opposite ∠'s of cyclic quadrilateral are supplementary)
$= \frac{2\pi}{5}$

$\theta = 2 \times \frac{2\pi}{5}$ (∠ at centre twice ∠ at circumference standing on same arc)
$= \frac{4\pi}{5}$

④ D

A(4, −2)   B(6, 2)

5 : −3

$$P = \left(\frac{5(6) - 3(4)}{5-3}, \frac{5(2) - 3(-2)}{5-3}\right)$$
$= (9, 8)$

⑤ B

$x = 3t$ ① $y = -6t^2$ ②

$t = \frac{x}{3}$ ①

sub ① into ②

$$y = -6\left(\frac{x}{3}\right)^2$$
$$y = -\frac{2}{3}x^2$$
$$x^2 = -\frac{3}{2}y$$

⑥ A

$$P(\text{youngest not together}) = 1 - P(\text{youngest together})$$
$$= 1 - \frac{6! \times 2!}{7!}$$
$$= \frac{5}{7}$$

Note $(n-1)!$ for circular arrangements

⑦ D

$y = 3x - 1$ $m_1 = 3$

$x + 2y - 6 = 0 \rightarrow y = -\frac{1}{2}x + 3$ $m_2 = -\frac{1}{2}$

$$\tan\theta = \left|\frac{3 - -\frac{1}{2}}{1 + 3 \times -\frac{1}{2}}\right|$$
$$= |-7|$$
$$= 7$$

⑧ C

$y = \cos^{-1}x$ $-1 \le x \le 1$

$\therefore y = \cos^{-1}\left(\frac{3x}{2}\right)$ $-1 \le \frac{3x}{2} \le 1$

$-\frac{2}{3} \le x \le \frac{2}{3}$

⑨ A

$P(2) = 4$
$$a \times 2^3 - 5 \times 2^2 + 10 \times 2 + 12 = 4$$
$$8a + 12 = 4$$
$$8a = -8$$
$$a = -1$$
$$P(x) = -x^3 - 5x^2 + 10x + 12$$
$$P(-3) = -(-3)^3 - 5(-3)^2 + 10(-3) + 12$$
$$= -36$$

⑩ C

$$y = \cos(\ln x)$$
$$\frac{dy}{dx} = -\sin(\ln x) \times \frac{1}{x}$$
$$= \frac{-\sin(\ln x)}{x}$$

$\left\{ u = -\sin(\ln x),\ v = x;\ u' = \frac{-\cos(\ln x)}{x},\ v' = 1 \right\}$

$$\frac{d^2y}{dx^2} = \frac{x\left(\frac{-\cos(\ln x)}{x}\right) - -\sin(\ln x)}{x^2}$$
$$= \frac{\sin(\ln x) - \cos(\ln x)}{x^2}$$

(11) a) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= \left(\frac{-(-5)}{6}\right)^2 - 2\left(\frac{-13}{6}\right)$

$= \frac{181}{36}$ OR $5\frac{1}{36}$

b) P×1, A×4, R×2, M×1, T×2

N^o arrangements $= \frac{10!}{4!\times 2!\times 2!}$

$= 37\,800$

c) Critical Points

Equality

$\frac{5}{x-4} = x$

$5 = x^2 - 4x$

$x^2 - 4x - 5 = 0$

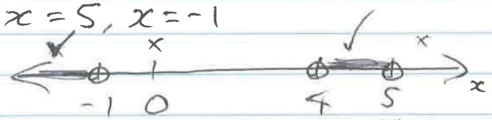
$(x-5)(x+1) = 0$

$x = 5,\ x = -1$

Zero Denom

$x - 4 = 0$

$x = 4$



$x < -1,\ 4 < x < 5$

d) $u = e^x$

$du = e^x dx$

$x = \ln\frac{1}{\sqrt{2}} \rightarrow u = e^{\ln\left(\frac{1}{\sqrt{2}}\right)} = \frac{1}{\sqrt{2}}$

$x = 0 \rightarrow u = e^0 = 1$

$\int_1^{\frac{1}{\sqrt{2}}} \frac{5}{\sqrt{1-u^2}}\,du = \left[5\sin^{-1}u\right]_1^{\frac{1}{\sqrt{2}}}$

$= 5\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - 5\sin^{-1}(1)$

$= 5\times\frac{\pi}{4} - 5\times\frac{\pi}{2}$

$= \frac{-5\pi}{4}$

(11) e) $\frac{d}{dx}\left(x^2\cos^{-1}x\right)$

$\begin{cases} u = x^2 & v = \cos^{-1}x \\ u' = 2x & v' = \frac{-1}{\sqrt{1-x^2}} \end{cases}$

$= 2x\cos^{-1}x - \frac{x^2}{\sqrt{1-x^2}}$

f) Power ratio required $= 1:3$

$= 3:9$

Term independent of x $= {}^{12}C_9(2x^3)^3\left(-\frac{3}{x}\right)^9$

$= {}^{12}C_9(2^3)(-3)^9$

$= -{}^{12}C_9(2^3)(3^9)$ OR $-{}^{12}C_3(2^3)(3^9)$

(12) a) i) $T = 160 - Ae^{-kt}$

when $t=0$, $T=-20$

$-20 = 160 - Ae^0$

$A = 180$

$\therefore T = 160 - 180e^{-kt}$

when $t=50$, $T=40$

$40 = 160 - 180e^{-50k}$

$180e^{-50k} = 120$

$e^{-50k} = \frac{2}{3}$

$e^{50k} = \frac{3}{2}$

$50k = \ln\left(\frac{3}{2}\right)$

$k = \frac{1}{50}\ln\left(\frac{3}{2}\right)$

ii) $T = 160 - 180e^{-\frac{1}{50}\ln\left(\frac{3}{2}\right)\times t}$

when $T = 60$

$60 = 160 - 180e^{-\frac{1}{50}\ln\left(\frac{3}{2}\right)t}$

$180e^{-\frac{1}{50}\ln\left(\frac{3}{2}\right)t} = 100$

$e^{-\frac{1}{50}\ln\left(\frac{3}{2}\right)t} = \frac{5}{9}$

$t = \dfrac{\ln\left(\frac{5}{9}\right)}{-\frac{1}{50}\ln\left(\frac{3}{2}\right)}$

$= 72\cdot48\ldots$ mins

$= 72$ OR 73 (accept either)

b) $P(HH) = \frac{1}{4}$ $\quad P(\text{other}) = \frac{3}{4}$

$\left(\frac{1}{4} + \frac{3}{4}\right)^{10}$

$P = {}^{10}C_6\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)^6$ OR ${}^{10}C_4\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^4$

(12) c) i) $\angle ABD = \frac{\theta}{2}$ (Angle at centre $2\times\angle$ at circumference standing on same arc)

$\angle DBY = 180° - \frac{\theta}{2}$ (straight $\angle DBY = 180°$)

ii) Similarly $\angle ACY = 180° - \frac{\theta}{2}$

$180° - \frac{\theta}{2} + 180° - \frac{\theta}{2} + \angle BXC + \angle BYC = 360°$ ($\angle$ sum of quadrilateral BXCY)

$360° - \theta + \angle BXC + \angle BYC = 360°$

$\therefore \angle BXC + \angle BYC = \theta$

d) $V = \pi\int_0^{\pi}\left(\cos\frac{x}{2}\right)^2 dx$

$= \pi\int_0^{\pi}\cos^2\frac{x}{2}\, dx$

$= \pi\int_0^{\pi}\frac{1}{2} + \frac{1}{2}\cos x\, dx$

$= \pi\left[\frac{x}{2} + \frac{1}{2}\sin x\right]_0^{\pi}$

$= \pi\left(\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right) - \left(\frac{0}{2} + \frac{1}{2}\sin 0\right)\right)$

$= \frac{\pi^2}{2}$ units^3

(12) e) Prove for $n=2$

$LHS = 2^2 = 4$ $\qquad$ $RHS = 2\times 2 = 4$

$\therefore LHS \geq RHS$

$\therefore$ true for $n=2$

Assume true for $n=k$

ie assume $k^2 \geq 2k$

Prove true for $n=k+1$

RTP $(k+1)^2 \geq 2(k+1)$

$LHS = (k+1)^2 = k^2+2k+1$ $\qquad$ $RHS = 2(k+1) = 2k+2$

Now $k^2+2k+1 \geq 2k+2k+1$ by assumption

but $2k+2k+1 \geq 2k+2$ as $k \geq 2$

$\therefore k^2+2k+1 \geq 2k+2$

$LHS \geq RHS$

$\therefore$ True for $n=k+1$

$\therefore$ True by mathematical induction for all integers $n \geq 2$

(13) a) i) $f(x) = \dfrac{2x^2+9}{x^2+4}$

$= \dfrac{2(x^2+4)+1}{x^2+4}$

$= \dfrac{2(x^2+4)}{x^2+4} + \dfrac{1}{x^2+4}$

$= 2 + \dfrac{1}{x^2+4}$

OR $2 + \dfrac{1}{x^2+4} = \dfrac{2(x^2+4)}{x^2+4} + \dfrac{1}{x^2+4}$

$= \dfrac{2x^2+8+1}{x^2+4}$

$= \dfrac{2x^2+9}{x^2+4}$

$= f(x)$

ii) $y=2$

iii) $f'(x) = \dfrac{-2x}{(x^2+4)^2}$

when $f'(x)=0$

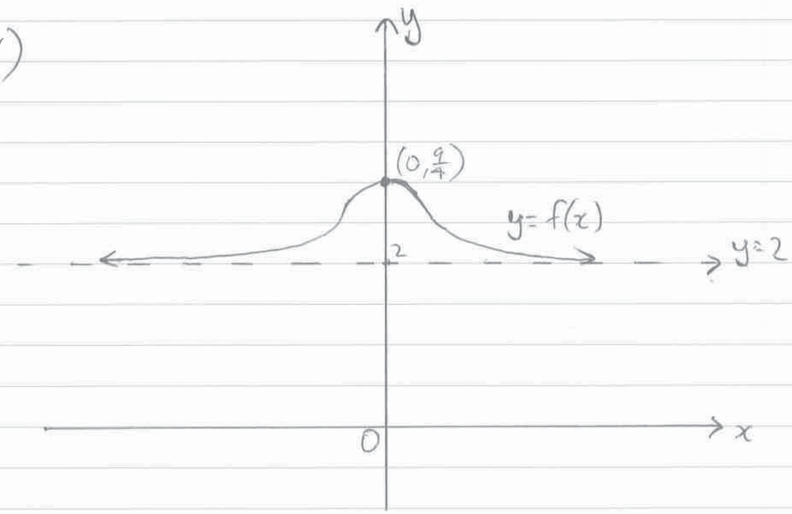
$-2x=0$

$x=0$

$f(0) = \dfrac{2\times 0^2+9}{0^2+4} = \dfrac{9}{4}$

$\therefore \left(0, \frac{9}{4}\right)$

iv)



(13) a) v) Area $= 2\int_0^2 \left(2 + \frac{1}{x^2+4}\right) dx$

$= 2\left[2x + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^2$

$= 2\left(\left(2(2) + \frac{1}{2}\tan^{-1}\left(\frac{2}{2}\right)\right) - \left(2(0) + \frac{1}{2}\tan^{-1}\left(\frac{0}{2}\right)\right)\right)$

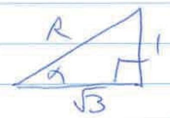
$= 2\left(4 + \frac{1}{2} \times \frac{\pi}{4}\right)$

$= 8 + \frac{\pi}{4}$ units^2

(13) b) i) $R\sin(2t+\alpha) = R\sin 2t\cos\alpha + R\cos 2t\sin\alpha$

$\therefore R\cos\alpha = \sqrt{3}$ $\qquad R\sin\alpha = 1$

$\cos\alpha = \frac{\sqrt{3}}{R}$ $\qquad \sin\alpha = \frac{1}{R}$



$R^2 = 1^2 + (\sqrt{3})^2$

$R = \sqrt{4}$

$= 2$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$\therefore x = 2\sin\left(2t + \frac{\pi}{6}\right)$

ii) $\dot{x} = 4\cos\left(2t + \frac{\pi}{6}\right)$

$\ddot{x} = -8\sin\left(2t + \frac{\pi}{6}\right)$

$= -4 \times 2\sin\left(2t + \frac{\pi}{6}\right)$

$= -4x$

$= -2^2 x$ $\qquad \therefore$ In form $\ddot{x} = -n^2 x$ with $n = 2$

iii) When $\dot{x} = 0$

$0 = 4\cos\left(2t + \frac{\pi}{6}\right)$

$2t + \frac{\pi}{6} = \frac{\pi}{2}$

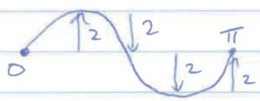
$2t = \frac{\pi}{3}$

$t = \frac{\pi}{6}$ sec

$x = 2\sin\left(2 \times \frac{\pi}{6} + \frac{\pi}{6}\right)$

$= 2$

iv) Period $= \pi$ and $a = 2$



(It goes through a full cycle in π seconds $\therefore$ the $\frac{\pi}{6}$ is irrelevant for distance)

Distance travelled $= 4 \times 2$

$= 8$ m

(14) a) Let $f(x) = x^3 - \cos x$

$f'(x) = 3x^2 + \sin x$

$x_2 = 0\cdot 9 - \dfrac{0\cdot 9^3 - \cos 0\cdot 9}{3(0\cdot 9)^2 + \sin 0\cdot 9}$

$= 0\cdot 8665\ldots$

$\doteqdot 0\cdot 87$

b) $4\sin\frac{x}{2}\cos\frac{x}{2} = \sqrt{3}$

$2\sin\frac{x}{2}\cos\frac{x}{2} = \dfrac{\sqrt{3}}{2}$

$\sin x = \dfrac{\sqrt{3}}{2}$

$x = n\pi + (-1)^n \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= n\pi + (-1)^n \times \dfrac{\pi}{3}$ where n is an integer

c) $y^2 = 13^2 - x^2$

$y = \sqrt{169 - x^2}$

$\dfrac{dy}{dx} = \frac{1}{2}(169 - x^2)^{-\frac{1}{2}} \cdot -2x$ $\qquad \dfrac{dx}{dt} = -0\cdot 1$

$= \dfrac{-x}{\sqrt{169 - x^2}}$

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$

$= \dfrac{-5}{\sqrt{169 - 5^2}} \times -0\cdot 1$

$= \dfrac{1}{24}\ ms^{-1}$

(14) d) i) $V = 25,\ g = 9\cdot 8$

$x = 25t\cos\theta$ $\qquad y = -\frac{1}{2} \times 9\cdot 8 \times t^2 + 25t\sin\theta$

$t = \dfrac{x}{25\cos\theta}$ (1) $\qquad = -4\cdot 9t^2 + 25t\sin\theta$ (2)

sub (1) into (2)

$y = -4\cdot 9\left(\dfrac{x}{25\cos\theta}\right)^2 + 25\left(\dfrac{x}{25\cos\theta}\right) \cdot \sin\theta$

$= \dfrac{-4\cdot 9x^2}{625\cos^2\theta} + x\tan\theta$

$= \dfrac{-49x^2}{6250}\sec^2\theta + x\tan\theta$

$= \dfrac{-49x^2}{6250}(1 + \tan^2\theta) + x\tan\theta$

ii) Let $x = 60$ & $y = 1\cdot 2$

$1\cdot 2 = \dfrac{-49(60)^2}{6250}(1 + \tan^2\theta) + 60\tan\theta$

$1\cdot 2 = \dfrac{-3528}{125} - \dfrac{3528}{125}\tan^2\theta + 60\tan\theta$

$\dfrac{3528}{125}\tan^2\theta - 60\tan\theta + \dfrac{3678}{125} = 0$

$3528\tan^2\theta - 7500\tan\theta + 3678 = 0$

$1764\tan^2\theta - 3750\tan\theta + 1839 = 0$

$\tan\theta = \dfrac{3750 \pm \sqrt{(-3750)^2 - 4(1764)(1839)}}{2 \times 1764}$

$= 1\cdot 3583\ldots,\ 0\cdot 767\ldots$

$\theta = 53\cdot 64\ldots^\circ,\ 37\cdot 505\ldots^\circ$

$\doteqdot 53^\circ 38',\ 37^\circ 30'$

$\therefore\ 37^\circ 30' < \theta < 53^\circ 38'$

(Accept $38^\circ < \theta < 54^\circ$)

(14) e) i) $^nC_0x^{-1} + {}^nC_1 + {}^nC_2x + {}^nC_3x^2 + {}^nC_4x^3 + \ldots + {}^nC_nx^{n-1}$

$$= \frac{{}^nC_0 + {}^nC_1x + {}^nC_2x^2 + {}^nC_3x^3 + {}^nC_4x^4 + \ldots + {}^nC_nx^n}{n}$$

$$= \frac{(1+x)^n}{x}$$

ii) $\frac{(1+x)^n}{x} = {}^nC_0x^{-1} + {}^nC_1 + {}^nC_2x + {}^nC_3x^2 + {}^nC_4x^3 + \ldots + {}^nC_nx^{n-1}$

Differentiating both sides

$$\frac{xn(1+x)^{n-1} - (1+x)^n}{x^2} = -{}^nC_0x^{-2} + {}^nC_2 + 2\,{}^nC_3x + 3\,{}^nC_4x^2 + \ldots + (n-1)\,{}^nC_nx^{n-2}$$

When $x = 1$

$n(2)^{n-1} - 2^n = -{}^nC_0 + {}^nC_2 + 2\,{}^nC_3 + 3\,{}^nC_4 + \ldots + (n-1)\,{}^nC_n$

$n(2)^{n-1} - 2 \times (2)^{n-1} = -1 + {}^nC_2 + 2\,{}^nC_3 + 3\,{}^nC_4 + \ldots + (n-1)$

$2^{n-1}(n-2) = {}^nC_2 + 2\,{}^nC_3 + 3\,{}^nC_4 + \ldots + (n-2)\,{}^nC_{n-1} + (n-2)$

$(n-2)(2^{n-1} - 1) = {}^nC_2 + 2\,{}^nC_3 + 3\,{}^nC_4 + \ldots + (n-2)\,{}^nC_{n-1}$